Frequency Dependent Krylov Vectors for Second-Order Damped Systems

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(Received December 10, 1998)

An algorithm for the model reduction of a damped structural dynamics system is developed by combining parameter matching technique with a spectrum shifting. In the procedure, there is no complex algebra and no system order increment unlike the conventional eigenvalue computation method involved with damping effect. By taking starting vectors relevant to actuator and sensor locations, the effect of excitation and measurement is strategically accounted. With the selection of loading frequency as a shifting parameter, the proposed algorithm shows more accurate dynamic response than the existing method with respect to the full-order model. It is shown that obtaining a reduced-order model should include not only low frequency moments but also high frequency moments of the full-order system. Two examples are employed to illustrate the efficacy of the proposed algorithm.

Key Words : Spectrum Shifting, Krylov Vectors, Damped Eigenproblem, Dynamic Response

1. Introduction

One of the main issues in modeling of flexible structures or any other large-scale system is the dimensionality of systems, especially those formulated by the finite element method. The approach can lead to accurate modeling, but a high order system is obtained. Hence, a model reduction approach plays an important role for efficient dynamic analysis. In the model reduction approach, the selection of a projection basis is important to the accuracy of the reduced model. Many authors have investigated basis selection by eigenmodes, static modes in component mode synthesis and Krylov vectors (which can be considered as static modes). Especially, Krylov vectors have been used in the application to eigenvalue analysis and to structural dynamics model reduction problems. Using the Krylov vectors, several algorithms have been developed, such as the Wilson method (Wilson, and Dickens, 1982) and the Lanczos algorithm (Chen and Taylor, 1988 ; Nour-Onid and Clough, 1985).

Su and Craig (1991) presented a model reduction algorithm based on the combination of Krylov vectors and a parameter matching concept without destroying the symmetry and physical meaning of the damped system matrices. Furthermore, the reduced-order model has the valuable property of parameter matching. However, their algorithm cannot effectively account for specific parameters and frequency contents due to the matching of low frequency modes. This is because the Krylov sequence (Cook et al., 1989) converges to the lowest mode shape. As a result, their method generates vectors that are close to a few lowest mode shapes. Therefore, their method is most effective when the external input frequency is relatively low. In practice, many types of external inputs have a significant frequency content or a specific frequency content. It is common that lower modes are frequently employed as basis vectors to serve as an effective system model by normal modes or Krylov vectors. However, both methods fail to account for the frequency contents of loadings. Specifically, low frequency matching

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with Krylov vectors cannot capture specific frequency modes which may strongly influence the response. Harmonic loads may be, for example, produced by an earthquake. Skelton and Yousuff (1983) showed that certain higher mode shapes are important in the application of his modal reduction method to large flexible structures. When higher frequencies in loadings predominate, Rayleigh-Ritz methods may become ineffective. This is because loadings with a frequency content tend to excite the mode shapes of systems.

By including the frequency dependent vectors in the processing of vector generation, the accuracy of dynamic analysis is dramatically improved (Joo et al., 1989; Sandridge and Haftka, 1991). Xia and Humar (1992) motivated by the reference (Joo et al; 1989) present an algorithm in order to account for the frequency content of the loading, a parameter that may strongly influence the response, particularly for loading with a high frequency content, so that the original Ritz vectors algorithm is improved in the sense of selecting more essential modes. However, those algorithms deal with undamped linear systems and destroy the symmetry and physical meaning of the system matrices because it is necessary to put a second-order matrix differential equation into first-order form.

A contribution of this paper is a model reduction algorithm that can account for parameters in low frequency content as well as in high-frequency content for a general second-order damped system together with the influence of actuator and sensor locations. In order to capture both parameters simultaneously, a spectrum shifting strategy is employed. The spectrum shifting strategy (Bathe, 1982) has long been used in the extraction of mode shapes and frequencies. By utilizing the shifting strategy, both low and high frequency parameters can be effectively extracted, which is the main difference between this method and the Su and Craig's. In other words, a distinguishing feature of this new model reduction method is to locate a certain frequency parameter dynamically rather than statically within the dominant range of frequencies. Furthermore, the algorithm can be applied to the dynamic analysis of structures in civil engineering.

In the following sections, the parameter matching technique with Krylov sequence is first reviewed. Second, the algorithm of frequency dependent Krylov vectors is proposed by incorporating spectrum shifting based on the reference (Su and Craig, 1991). Third, the efficiency and accuracy of the impulse responses and dynamic responses with regard to harmonic loading is numerically demonstrated with the application to two truss structures characterized by closely spaced modes. Finally, a conclusion has been made.

2. Model Order Reduction by Matching System Parameters

Using the parameter matching method, a model reduction algorithm was introduced by Villemagne and Skelton (1987) for an n-th first order linear time-invariant system. The reduced-order model is obtained by an oblique projection approach through high-frequency moments or low-frequency moments which are obtained by Laurent or Taylor series expansion of output frequency response. This reduction approach can preserve the necessary system parameters. Recently, Su and Craig (1991) proposed a combined algorithm of Krylov sequence and low-frequency matching method. In the following section, the basic procedure is reviewed.

2.1 General linear systems

An nth order, linear, time-invariant system is expressed by

$$\dot{z}=Az+Bu, \quad z\in R^n, \quad u\in R^l \tag{1}$$

$$y=Cz, \quad y\in R^m \tag{2}$$

for which the transfer function $G(s)=C(sI-A)^{-1}B$ can be formally expanded in a Larant series around $s=\infty$ as

$$G(s)=\sum_{i=0}^{\infty} CA^iBs^{-i-1} \tag{3}$$

If the system has no pole at the origin, then the Taylor series expansion of $G(s)$ around $s=0$ yields

$$G(s)=\sum_{i=0}^{\infty} -CA^{-i-1}Bs^i \tag{4}$$

From Eq. (3), we get a set of system parameters $\{CA^iB|i=0, 1, \cdots\}$, which are termed Markov parameters or high-frequency moments (Villemagne and Skelton, 1987). We get another set of system parameters $\{CA^{-i}B|i=1, 2, \cdots\}$, which are termed time moments or low-frequency moments (Villemagne and Skelton, 1987). These two sets of parameters constitute pieces of system data for the triple (A, B, C). Villemagne and Skelton (Villemagne and Skelton, 1987) provide a toolbox for producing parameter matching reduced-order models. The reduced-order model is obtained by an oblique projection approach and is in the form

$$\dot{z}_R = A_R z_R + B_R u, \quad y = C_R z_R \tag{5}$$

where $r<n$, $z_R \in R^r$, $A_R = TAR$, $B_R = TB$, $C_R = CR$, and $TR = I_r$, with T and R, the left and right projection matrices. It is shown in Ref. (Villemagne and Skelton, 1987) that T and R are chosen such that span $[T]$=span $[(A^T)^{-p}C^T, (A^T)^{-p+1}C^T, \cdots, (A^T)^q C^T]$ and span$[R]$=span $[A^{-s}B, A^{-s+1}B, \cdots, A^t B]$ with $p, q, s, t \geq 0$ and $p+q=s+t$, then the reduced-order model matches $p+s$ low-frequency moments and $q+t$ high-frequency moments. That is, $C_R A^i_R B_R = CA^iB$, for $i=-p-s, \cdots, q+t$.

2.2 Structural Dynamics Systems

A structural dynamics equation can be described by the input-output relation

$$M\ddot{x} + C\dot{x} + Kx = Pu \tag{6}$$

$$y = Vx + W\dot{x} \tag{7}$$

where $x \in R^n$ is the displacement vector; $u \in R^l$ the input vector: $y \in R^m$ the output measurement vector; M, C, and K the mass, damping, and stiffness matrices, respectively; and V and W the displacement and velocity sensor distribution matrices. The damping matrix is assumed to be symmetric. The frequency response solution of Eq. (6) is

$$X(\omega) = (K + j\omega C - \omega^2 M)^{-1} PU(\omega) \tag{8}$$

where $X(\omega)$ and $U(\omega)$ are the Fourier transforms of x and u. If the system is assumed to have no rigid-body modes, the output frequency response can be represented by a Taylor series;

$$\begin{aligned} Y(\omega) &= (V + j\omega W)(-\omega^2 M + j\omega C + K)^{-1} PU(\omega) \\ &= (V + j\omega W)(I + j\omega K^{-1}C - \omega^2 K^{-1}M)^{-1} K^{-1} PU(\omega) \\ &= \sum_{i=0}^{\infty} (V + j\omega W)(\omega^2 K^{-1}M - j\omega K^{-1}C)^i K^{-1} PU(\omega) \\ &= \{VK^{-1}P + j\omega(W - VK^{-1}C)K^{-1}P + \omega^2[VK^{-1}M \\ &\quad + WK^{-1}C - V(K^{-1}C)^2]K^{-1}P + \cdots\}. \end{aligned} \tag{9}$$

The low-frequency moments are defined as the coefficient matrices in the expansion series in Eq. (9).

In order to construct a reduced-order model that matches low-frequency moments, it is convenient that the first-order formulation is sought which is equivalent to Eq. (6).

$$\hat{M}\dot{z} + \hat{K}z = \hat{P}u \tag{10}$$

$$y = \hat{V}z \tag{11}$$

with

$$\hat{M} = \begin{bmatrix} C & M \\ M & 0 \end{bmatrix}, \quad \hat{K} = \begin{bmatrix} K & 0 \\ 0 & -M \end{bmatrix}$$

$$\hat{P} = \begin{bmatrix} P \\ 0 \end{bmatrix}, \quad \hat{V} = [V \quad W] \tag{12}$$

where $z = \{x, \dot{x}\}^T$. Now, the system equation is described by state-space form

$$\dot{z} = -\hat{M}^{-1}\hat{K}z + \hat{M}^{-1}\hat{P}u \tag{13}$$

$$y = \hat{V}z \tag{14}$$

Recalling that $CA^{-i}B$ is a low-frequency moment for a system expressed by Eqs. (1) and (2), we can describe the low-frequency moments from the first term in the last Eq. (9) for the system of Eqs. (13) and (14) as

$$\begin{aligned} T_i &= \hat{V}(-\hat{M}^{-1}\hat{K})^i \hat{M}^{-1}\hat{P} \\ &= (-1)^i \hat{V}(\hat{K}^{-1}\hat{M})^{i-1}\hat{K}^{-1}\hat{P}, \ i=1, 2, \cdots. \end{aligned} \tag{15}$$

It can be shown that $\hat{V}(\hat{K}^{-1}\hat{M})^i \hat{M}^{-1}\hat{P}$ is equal to the coefficient matrix associated with the $(j\omega)^{i-1}$ term in Eq. (9). For generating the projection subspace, Eq. (12) is substituted into Eq. (15).

$$T_i = (-1)^i [VW] \begin{bmatrix} -K^{-1}C & -K^{-1}M \\ I & 0 \end{bmatrix}^{i-1} \begin{bmatrix} K^{-1}P \\ 0 \end{bmatrix} \tag{16}$$

Using Eq. (16), a Krylov recurrence procedure is obtained and is described as

$$\begin{Bmatrix} Q^d_{j+1} \\ Q^v_{j+1} \end{Bmatrix} = \begin{bmatrix} -K^{-1}C & -K^{-1}M \\ I & 0 \end{bmatrix} \begin{Bmatrix} Q^d_j \\ Q^v_j \end{Bmatrix} \tag{17}$$

$$Q^d_{j+1} = -K^{-1}CQ^d_j - K^{-1}MQ^d_{j-1} \tag{18}$$

$$Q^v_{j+1} = Q^d_j. \tag{19}$$

Superscripts d and v denote displacement and velocity portions of the vector, respectively. Eq. (17) is used recursively for the vector generation (Su and Craig, 1991). Su and Craig (1991) claimed that the transformed system equation in Krylov coordinates shows dynamic spillover.

3. Frequency Dependent Krylov Vectors

The algorithm for generating low and high frequency dependent Krylov vectors is developed based upon the previous section (Su and Craig, 1991). The Krylov sequence (Cook et al., 1989) in the structural area is commonly employed for vector generation for undamped second-order differential systems. The Krylov sequence is written as

$$S_{undamped}=[r,\ Dr,\ D^2r,\ \cdots,\ D^m r] \qquad (20)$$

where $D=K^{-1}M$ and r is any non-zero vector. This sequence converges to the lowest mode shape. As a result, the sequence generates vectors that are close to the lowest mode shapes. This is the reason that the Su and Craig method have only matched low frequency modes. It is not sufficient to use only low frequency parameters to account for realistic problems. Therefore, some high frequency parameters are also necessary and this deficiency of Krylov sequence should be resolved. Furthermore, the reduced system no longer possesses the parameter matching property if the Krylov vectors generated for the undamped system are applied to the damped system.

In the following discussion, an efficient set of Krylov vectors should include shapes that are close to the mode shapes of the system with system parameters in the neighborhood of the predominant system parameters. It is well recognized that a Krylov sequence formed by using a shifted stiffness matrix converges to an eigenvector whose eigenvalue is closest to the shift. Hence, a Krylov sequence with respect to Eq. (10) is represented by

$$S_{damped}=[q,\ D_\sigma q,\ D_\sigma^2 q,\ \cdots,\ D_\sigma^m q] \qquad (21)$$

where q is any non-zero vector (which is commonly a static correction vector)

$$D_\sigma=(\hat{K}-\sigma\hat{M})^{-1}\hat{M} \qquad (22)$$

where σ represents the shift. It is numerically convenient that Eq. (21) is expressed by the inverse iteration formulation which has been used for eigenvector and eigenvalue computation for an undamped system. In order to obtain an iterative formulation, the following equation,

$$z(t)=Qe^{(\lambda-\sigma)t} \qquad (23)$$

where λ is the eigenvalue and Q the eigenvector, is substituted into each state variable of $z(t)$ in Eq. (10). As a basic relation of the vector iteration method (Bathe, 1982), the eigenvalue problem (Chen and Taylor, 1988) can be formulated as

$$(\lambda-\sigma)\begin{bmatrix} C+2\sigma M & M \\ M & 0 \end{bmatrix}\begin{Bmatrix} Q \\ (\lambda-\sigma)Q \end{Bmatrix}= \begin{bmatrix} -K-\sigma C-\sigma^2 M & 0 \\ 0 & M \end{bmatrix}\begin{Bmatrix} Q \\ (\lambda-\sigma)Q \end{Bmatrix} \qquad (24)$$

where the shifted matrices in both sides remain symmetric. From the above eigenproblem, an iteration formula by the inverse iteration technique with Eq. (17) and (Villemagne and Skelton, 1987) is represented as

$$\begin{bmatrix} -K-\sigma C-\sigma^2 M & 0 \\ 0 & M \end{bmatrix}\begin{Bmatrix} Q_{j+1}^d \\ Q_{j+1}^v \end{Bmatrix}=\begin{bmatrix} C+2\sigma M & M \\ M & 0 \end{bmatrix}\begin{Bmatrix} Q_j^d \\ Q_j^v \end{Bmatrix}. \qquad (25)$$

Here, with decomposition and rearrangement, a recursive formula is proposed without increasing system order as

$$Q_{j+1}^d=[-K-\sigma C-\sigma^2 M]^{-1}\{[C+2\sigma M]Q_j^d+MQ_j^v\} \qquad (26)$$

$$Q_{j+1}^v=Q_j^d \qquad (27)$$

It is apparent that vectors taken from the sequence in Eqs. (26) and (27) can generate efficient vectors provided σ is selected to represent the concerned parameter. With the above formulation, it is beneficial to keep the symmetry of the system after the spectrum shifting operation and to reduce numerical computation without using $\hat{K}$ and $\hat{M}$ to precede each sequence. This formula is used for the generation of frequency dependent Krylov vectors. The entire algorithm is presented in Table 1. In the algorithm, starting vectors are chosen to produce a K-normalized $Q^{n\times(s\ blocks)}$ matrix and a transformed system equation with force and sensor distribution matrices having nonzero elements only in the first block. The size of the block can be reduced if the actuator/sensor location is collocated, since $\hat{P}$ is the linearly independent portion of $\bar{P}=[P, V^T, W^T, (M^{-1}W^T)]$. There are several techniques for the selection of independent vectors, such as singular value decomposition or a row-searching method (Brogan, 1991). The first part of the Krylov sequence is similar to the Su and Craig method, but the second part of the Krylov sequence employs Eqs. (26) and (27) to capture the desired parameters. The final set of vectors is used for a system to be reduced. Unlike the first-order state space formulation for model reduction, we have a second-order reduced system which is physically meaningful so that mass, damping, and spring matrices are classified. The transformed system equation is expressed by

$$Q^TMQ\ddot{\hat{x}}+Q^TCQ\dot{\hat{x}}+Q^TKQ\hat{x}=Q^TPu \quad (28)$$

$$y=VQ\hat{x}+WQ\dot{\hat{x}}. \quad (29)$$

The transformation can be represented as

$$x=Q\hat{x} \quad (30)$$

where $Q^{n\times(s\ blocks)}$ is formed from the vectors generated by the proposed algorithm; x are the physical coordinates; $\hat{x}$ are the transformed coordinates. Now, the reduced system can be used for dynamic analysis of large flexible structures. The matrices of the transformed system equation are not diagonalized, but it is efficient in the aspects of computational speed, system memory.

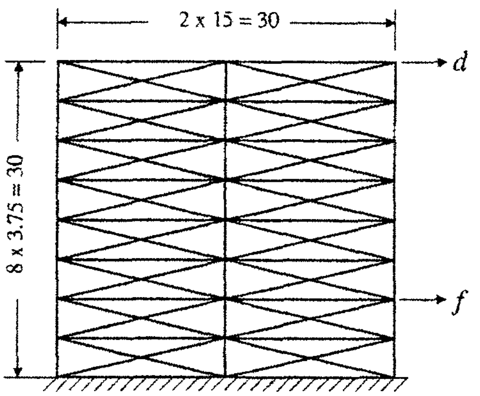
In the numerical simulation, the final set of vectors is orthogonalized to ensure the orthogonality between each vector by singular value decomposition. Of course, this does not change any results on the eigenvalues and eigenvectors.

Table 1 Algorithm of frequency dependent krylov vectors.

Operation & Calculation	
independent vector selection	$\hat{P}=[P, V^T, W^T, M^{-1}W^T]$
	$R_0^d=K^{-1}\hat{P}$
	$R_0^v=-M^{-1}W^T$
singular value decomposition	$U_0S_0U_0^T=\mathrm{svd}\{(R_0^d)^TKR_0^d\}$
first vectors	$Q_1^d=R_0^dU_0S_0^{1/2}$
	$Q_1^v=R_0^vU_0S_0^{1/2}$
calculate additional vectors	starting with $j=2, 3, \cdots, k-1$
	$R_j^d=-K^{-1}CQ_j^d-K^{-1}MQ_j^v$
	$R_j^v=Q_j^d$
orthogonalization	$R_j^d=R_j^d-\sum_{i=1}^{j}Q_i^d\{(Q_i^d)^TKR_j^d\}$
	$R_j^v=R_j^v-\sum_{i=1}^{j}Q_i^d\{(Q_i^v)^TKR_j^d\}$
singular value decomposition	$U_jS_jU_j^T=\mathrm{svd}\{(R_j^d)^TKR_j^d\}$
(j+1)th vectors	$Q_{j+1}^d=R_j^dU_jS_j^{1/2}$
	$Q_{j+1}^v=R_j^vU_jS_j^{1/2}$
	end
calculate frequency dependent	starting with $j=k, k+1, \cdots, s$
Krylov vectors (FDKV)	$F=[-K-\sigma C-\sigma^2M]$
	$R_j^d=F^{-1}\{[C+2\sigma M]Q_j^d+MQ_j^v\}$
	$R_j^v=Q_j^d$
orthogonalization	$R_j^d=R_j^d-\sum_{i=1}^{j}Q_i^d\{(Q_i^d)^TKR_j^d\}$
	$R_j^v=R_j^v-\sum_{i=1}^{j}Q_i^d\{(Q_i^v)^TKR_j^d\}$
singular value decomposition	$U_jS_jU_j^T=\mathrm{svd}\{(R_j^d)^TKR_j^d\}$
(j+1)th vectors	$Q_{j+1}^d=R_j^dU_jS_j^{1/2}$
	$Q_{j+1}^v=R_j^vU_jS_j^{1/2}$
	end
form the s-block projection matrix	$Q=[Q_1^v, Q_1^d, Q_2^d, \cdots, Q_s^d]$

4. Application to Plane Truss Models

Numerical examples are presented in order to compare the accuracy of dynamic responses. The system model in Fig. 1 is adopted from Su and Craig's paper (1991). The structure consists of 48 degrees of freedom, a force actuator f, and a displacement sensor d. The structure's geometry is designed to provide closely spaced eigenvalues. A formula (Craig, 1981) is used to provide a generalized proportional damping matrix such



ρA=1.6E+04 EA=8.0E+10

Fig. 1 Plane truss structure for numerical comparison.

Table 2 Eigenvalue comparison between models.

Full-Order System		Su and Craig Method (Su and Craig, 1991)		FDKV Method	
real	imaginary	real	imaginary	real	imaginary
-5.8057E-01	1.9343E+01	-5.8057E-01	1.9343E+01	-5.8058E-01	1.9344E+01
-1.7695E+00	5.8958E+01	-1.7695E+00	5.8958E+01	-1.7699E+00	5.8973E+01
-3.0517E+00	1.0167E+02				
-3.2728E+00	1.0904E+02	-3.2860E+00	1.0935E+02	-3.3005E+00	1.0964E+02
-4.2244E+00	1.4075E+02	-4.7513E+00	1.4370E+02		
-4.9120E+00	1.6366E+02			-5.4620E+00	1.5935E+02
-9.2901E+00	1.8557E+02				
-1.0217E+01	2.0409E+02	-9.3623E+00	1.9248E+02		
-1.0249E+01	2.0474E+02				
-1.1215E+01	2.2403E+02	-1.2186E+01	2.2254E+02		
-1.1791E+01	2.3553E+02	-1.6701E+01	2.3834E+02	-1.5969E+01	2.3519E+02
-1.7103E+01	2.4373E+02			-1.5970E+01	2.4244E+02
-1.7309E+01	2.4666E+02	-1.6423E+01	2.4639E+02		
-1.7351E+01	2.4727E+02				
-1.7470E+01	2.4896E+02				
-1.7592E+01	2.5071E+02				
-2.2742E+01	2.5166E+02				
-2. 2797E+01	2.5227E+02				
-2.3172E+01	2.5643E+02				
-2.3550E+01	2.6061E+02				
-2.5462E+01	2.8177E+02	-2. 6294E+01	2.7912E+02		
-3.2988E+01	2.9807E+02				
-3.3674E+01	3.0427E+02	-3.0042E+01	3.1150E+02	-2.7935E+01	3.0860E+02
-3.5697E+01	3.2255E+02				
-3.7307E+01	3.3709E+02			-3.1795E+01	3.3348E+02
-3.9020E+01	3.5258E+02				
-4.8840E+01	3.7250E+02				
-5.5251E+01	4.2140E+02				
-6.1027E+01	4.6546E+02				
-6.5348E+01	4.9841E+02			-6.2176E+01	4.9064E+02
-7.0248E+01	5.3578E+02				
-8.7874E+01	5.7920E+02			-7.2096E+01	5.8949E+02
-8.7909E+01	5.7943E+02				
-1.1614E+02	7.6550E+02				
-1.2220E+02	8.0546E+02				
-1.2428E+02	8.1918E+02				
-1.7008E+02	9.8594E+02				
-1.8212E+02	1.0557E+03				
-1.8489E+02	1.0717E+03				

that modes 1-5 have a 3% damping ratio, modes 6-10 have a 5% damping ratio, and the remaining higher order modes have successively higher damping. A finite element program was written in FORTRAN for dynamic analysis. The structure is reduced to 10 degrees of freedom by the Su and Craig method or by the new algorithm. In the new reduction procedure, 1 set of vectors is obtained by the Su and Craig method for load dependent Krylov vectors and 4 sets obtained by the FDKV method are generated with an eigenvalue magnitude of 240 rad/sec as a dominant frequency.

In Table 2, complex eigenvalues are computed according to both the full-order model and the reduced-order models. The complex eigenvalues from both reduction methods are closely placed to the eigenvalues of the full-order model. The reduced models have eigenvalues corresponding to actuator and sensor configurations so that skipped eigenvalues will not take part in dynamic response. As expected, the Su and Craig method converges to the lower frequency region, and generates modes only in the average sense around eigenvalue with magnitude of 240 rad/sec. However, the FDKV method shows that precise modes are located at eigenvalue magnitudes 235 rad/sec and 242 rad/sec. Higher modes can also be captured by the FDKV technique with the same order of the system as Su and Craig's method. It turns out that the Su and Craig method requires a larger order than the FDKV method in order to include higher modes.

In Figs. 2 and 3, the accuracy of the impulse response is compared for the full-order model and the reduced-order models by the Su and Craig method and the FDKV method. During 0.6 second dynamic response, the Su and Craig method shows high overshoot in the initial stage and has much error with respect to the full-order model in Fig. 3. However, the FDKV method shows less error than the Su and Craig method during 0.7 second simulation in Fig. 3.

As a second example in Fig. 4, the order 20 of flexible structure is reduced to the 10 degrees of freedom by the Su and Craig method and the FDKV method for the noncollocated actuator and sensor configuration. The proportional damping formula is chosen as $C=0.01M+0.01K$. In the proposed procedure, 1 set of vectors is given by the Su and Craig method and 4 sets, obtained by the FDKV method with the shifting parameter 1 as a dominant frequency, are generated. In Table 3, the eigenvalues of the full-order and the reduced-order models are compared. The impulse responses of the three models are compared in Fig 5. It is seen that the FDKV model and the full-order model have the same accuracy while the Su and Craig method is poor in Fig. 6.

As an example with the second plane truss

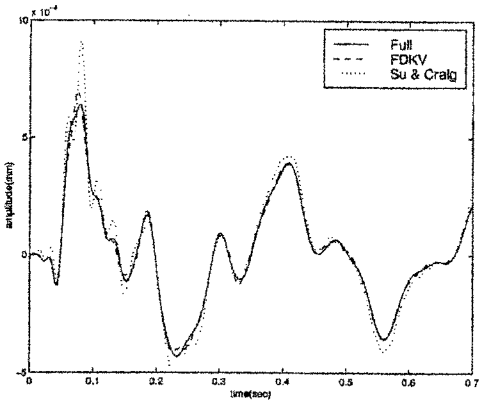


Fig. 2 Impulse response: full-order and reduced-order models.

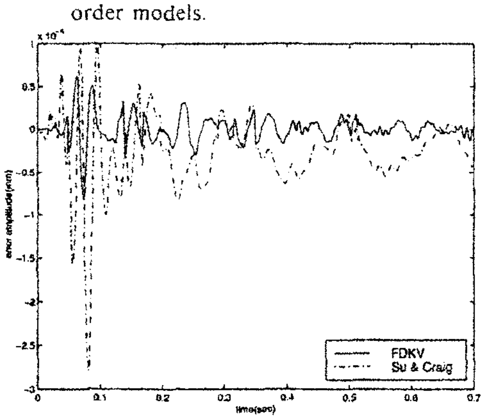


Fig. 3 Impulse response error with respect to full-order model.

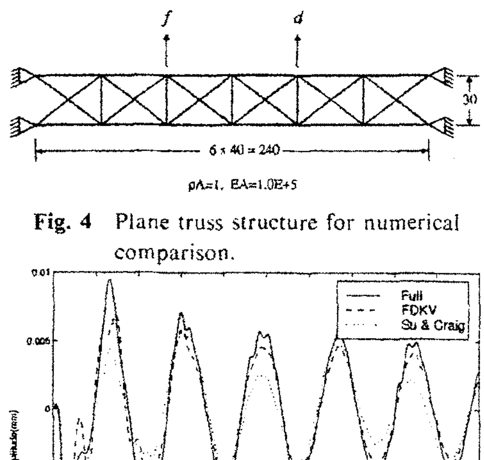


Fig. 4 Plane truss structure for numerical comparison.

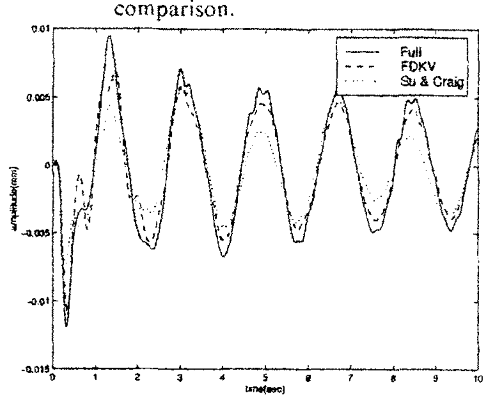


Fig. 5 Impulse response: full-order and reduced-order models.

Table 3 Eigenvalue comparison between models.

Full-Order System		Su and Craig Method		FDKV Method	
real	imaginary	real	imaginary	real	imaginary
−1.7621e−02	1.5887e+00i	−1.7621e−02	1.5887e+00i	−1.7621e−02	1.5887e+00i
−6.7536e−02	3.5359e+00i	−6.7539e−02	3.5360e+00i	−6.7536e−02	3.5359e+00i
−7.2702e−02	3.6790e+00i	−8.7245e−02	4.0548e+00i	−7.2724e−02	3.6796e+00i
−1.7297e−01	5.7935e+00i	−1.7372e−01	5.8064e+00i	−1.7308e−01	5.7954e+00i
−2.8840e−01	7.5231e+00i	−3.4450e−01	8.2330e+00i	−3.3418e−01	8.1070e+00i
−3.3375e−01	8.1017e+00i	−5.5535e−01	1.0477e+01i	−4.2476e−01	9.1527e+00i
−5.3943e−01	1.0324e+01i	−1.1388e+00	1.5016e+01i	−6.8908e−01	1.1677e+01i
−5.9768e−01	1.0871e+01i	−1.6160e+00	1.7877e+01i	−8.8112e−01	1.3208e+01i
−6.7985e−01	1.1598e+01i	−2.1378e+00	2.0542e+01i	−2.7154e+00	2.3124e+01i
−8.0699e−01	1.2639e+01i	−2.8655e+00	2.3746e+01i	−2.8488e+00	2.3678e+01i
−9.8344e−01	1.3954e+01i				
−1.1658e+00	1.5192e+01i				
−1.1817e+00	1.5295e+01i				
−1.2050e+00	1.5445e+01i				
−1.7528e+00	1.8614e+01i				
−1.7973e+00	1.8848e+01i				
−2.9362e+00	2.4034e+01i				
−3.0016e+00	2.4296e+01i				
−3.5316e+00	2.6322e+01i				
−4.0932e+00	2.8300e+01i				

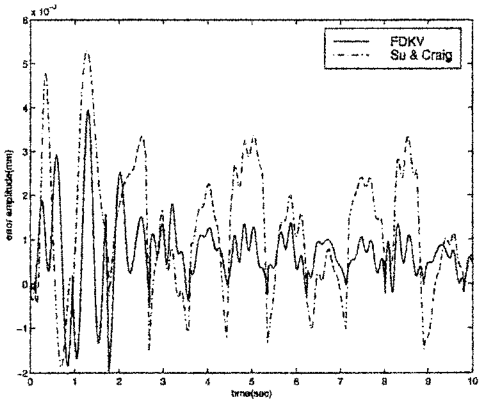


Fig. 6 Impulse response error with respect to Full-order model.

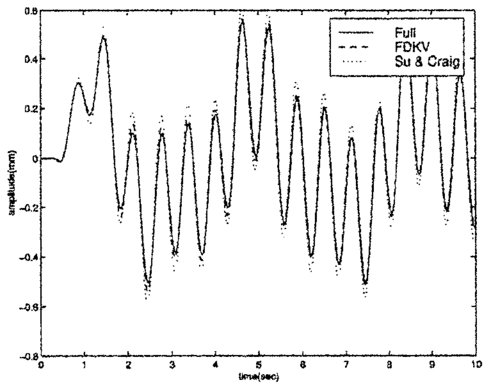


Fig. 7 Responses of sinusoidal input $\sin(10t)$: full-order and reduced-order models.

model to harmonic input, $\sin(10t)$, the shifting parameter σ is taken by 10 rad/sec to locate the dominant frequency of the harmonic input. Of course, it should avoid the resonance frequency for the selection of shifting parameter. In Figs. 7 and 8, the dynamic response by Su and Craig method is shown with large discrepancy with respect to the full-order model in both transient and steady state periods. As a result, the FDKV with the shifting parameter dynamically found the vectors depending on the excitation frequency.

5. Conclusions

The algorithm of frequency dependent Krylov vectors (FDKV) is presented for generating low and high frequency dependent vectors for second -order damped systems. In the generation procedure, the algorithm produces vectors influenced by actuator and sensor locations and provides an efficient method for model reduction of large

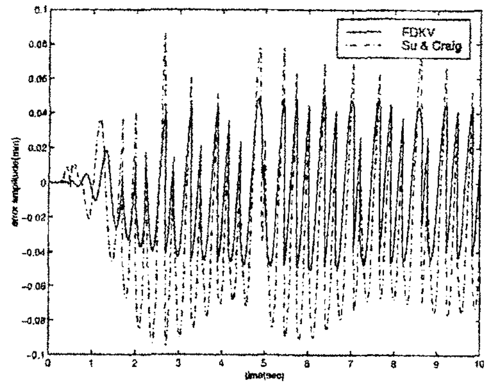


Fig. 8 Response errors of sinusoidal input $\sin(10t)$ with respect to full-order model.

flexible structures since it does not involved complex algebra used for conventional eigenvalue problems with damping effect. The existing method requires more modes to include higher frequency vectors, which means that the reduced system also becomes a high-order system so that model reduction effect will be diminished. In the new FDKV method, the combination of low parameter matching technique and spectrum shifting could provide a more efficient model reduction method for large flexible structures. Furthermore, it is shown that the FDKV method is better than the existing method in dynamic responses with respect to harmonic loading so that the spectrum shifting could be effectively used for general dynamic analysis of flexible structures.

As further researches, parametric studies are required to determine guidelines for selecting the optimum number of low and high frequency dependent vectors with regard to the configuration of actuators and sensors and input loading. In addition, a warning mechanism is necessary in the process of vector generation because the shifting parameter could correspond to a resonance frequency.

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